

FINALS WEEK!

MATH 34A

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On this worksheet are selected problems from homeworks 9 and 10 which were less done. Many of these problems highlight a number of key concepts from class, and in my opinion, would make excellent exam questions. I recommend you guys to try doing these on your own before asking me for help or working with others.

As before, I do not have any part in writing the final exam, nor did the professor have any part in giving these problems. I would imagine, though, that if you knew how to do these questions, you would be in pretty good shape for the exam (that is, you did the questions yourself and understand how to do them).

Good luck on your finals!

6.31 A plane flies at 200 mph for the first and last half hour of a flight. It flies at 400 mph the rest of the time. The route is 1000 miles long. The town of Erewhon is on the route 300 miles before the end. What is the distance of the plane from Erewhon after t hours of flying?

(a) $t \leq \frac{1}{2}$

(b) $\frac{1}{2} \leq t \leq 2$

(c) $2 \leq t \leq 2\frac{1}{2}$

(d) $2\frac{1}{2} \leq t \leq 3$

7.52 A commuter railway has 800 passengers per day and charges each one two dollars per day. For each 1 cents that the fare is increased, 5 fewer people will go by train. What is the greatest profit that can be earned.

7.55 What point on the graph $y = \sqrt{x}$ is closest to $(10, 0)$. (Hint: work out the square of the distance of a point on the curve from $(10, 0)$ and minimize the distance squared, this makes the algebra easier).

8.14 Beaker A contains 1 liter which is 20 percent oil and the rest is vinegar, thoroughly mixed up. Beaker B contains 2 liters which is 45 percent oil and the rest vinegar, completely mixed up. Half of the contents of B are poured into A, then completely mixed up. How much oil should now be added to A to produce a mixture which is 60 percent oil?

8.23 A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus, the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 50 ft, find the dimensions of the window so that the greatest possible amount of light is admitted. That is, find the base length and total height.

8.24 A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 20 km/h. Another boat has been heading due east at 10 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

9.27 A box has rectangular sides, top and bottom. The volume of the box is 3 cubic meters. The height of the box is half the width of the base. Express the total surface area of the box in terms of the height of the box.

If h is the height of the box, what is the surface area?

9.28 A tank of water has a base a circle of radius 2 meters and vertical sides. If water leaves the tank at a rate of 6 liters per minute, how fast is the water level falling in centimeters per hour? [1 liter is 1000 cubic centimeters]

9.35 The manager of a large apartment complex knows from experience that 90 units will be occupied if the rent is 324 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 1 dollar increase in rent. Similarly, one additional unit will be occupied for each 1 dollar decrease in rent. What rent should the manager charge to maximize revenue?

- 9.38 A manufacture has been selling 1750 television sets a week at \$510 each. A market survey indicates that for each \$13 rebate offered to a buyer, the number of sets sold will increase by 130 per week.
- (a) Find the function representing the demand $p(x)$, where x is the number of the television sets sold per week and $p(x)$ is the corresponding price.
 - (b) How large rebate should the company offer to a buyer, in order to maximize its revenue?
 - (c) If the weekly cost function is $148750 + 170x$, how should it set the size of the rebate to maximize its profit?

10.3 A street light is at the top of a 19 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 7 ft/sec along a straight path.

- (a) How fast is the tip of her shadow moving along the ground when she is 45 ft from the base of the pole?
- (b) How fast is the length of her shadow increasing?

10.5 A conical water tank with vertex down has a radius of 11 feet at the top and is 30 feet high. If water flows into the tank at a rate of $10\text{ft}^3/\text{min}$, how fast is the depth of the water increasing when the water is 14 feet deep?

10.14 As a preparation for the long bright summer days, Dr. Acula plans to store gourmet plasma in closed tin cans that have the shape of a cylinder with volume V . As an environmentally conscious member of the community, he wants to use as little metal as possible.

- (a) What is the height $h(r)$ for the can with minimum surface area in terms of the radius r of the bottom?
- (b) What is the minimum surface area for the can in terms of the portion size V ?